Top-Down Parsing

Recursive Descent Parser

- Consider the grammar:

  $S \to c\ A\ d$

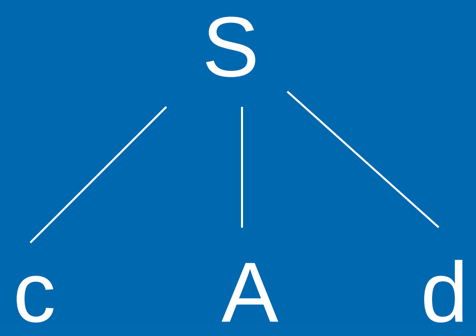
  $A \to ab \mid a$

  The input string is “*cad*”

Recursive Descent Parser (Cont.)

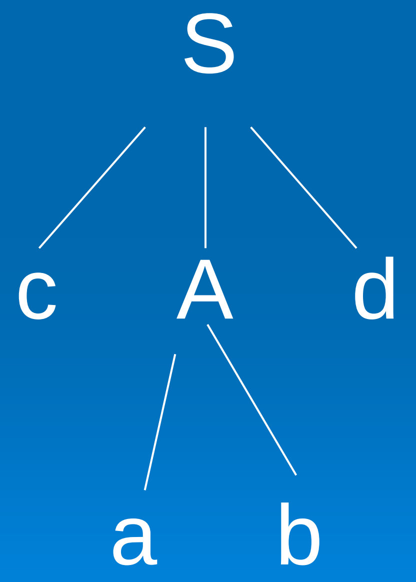
- Build parse tree:

step 1. From start symbol.



Recursive Descent Parser (Cont.)

Step 2. We expand A using the first alternative A → ab to obtain the following tree:



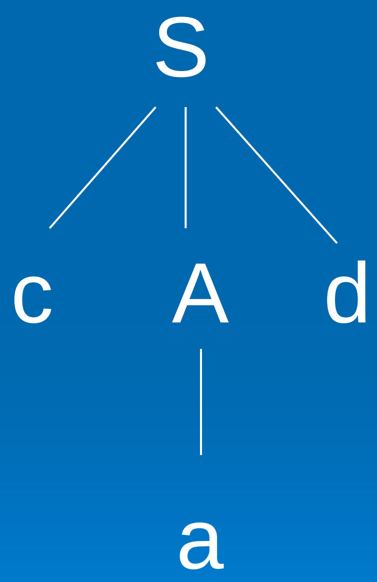
Recursive Descent Parser (Cont.)

- Now, we have a match for the second input symbol “a”, so we advance the input pointer to “d”, the third input symbol, and compare d against the next leaf “b”.

- Backtracking
  - Since “b” does not match “d”, we report failure and go back to A to see whether there is another alternative for A that has not been tried - that might produce a match!
  - In going back to A, we must reset the input pointer to “a”.

Recursive Descent Parser (Cont.)

Step 3.



Creating a top-down parser

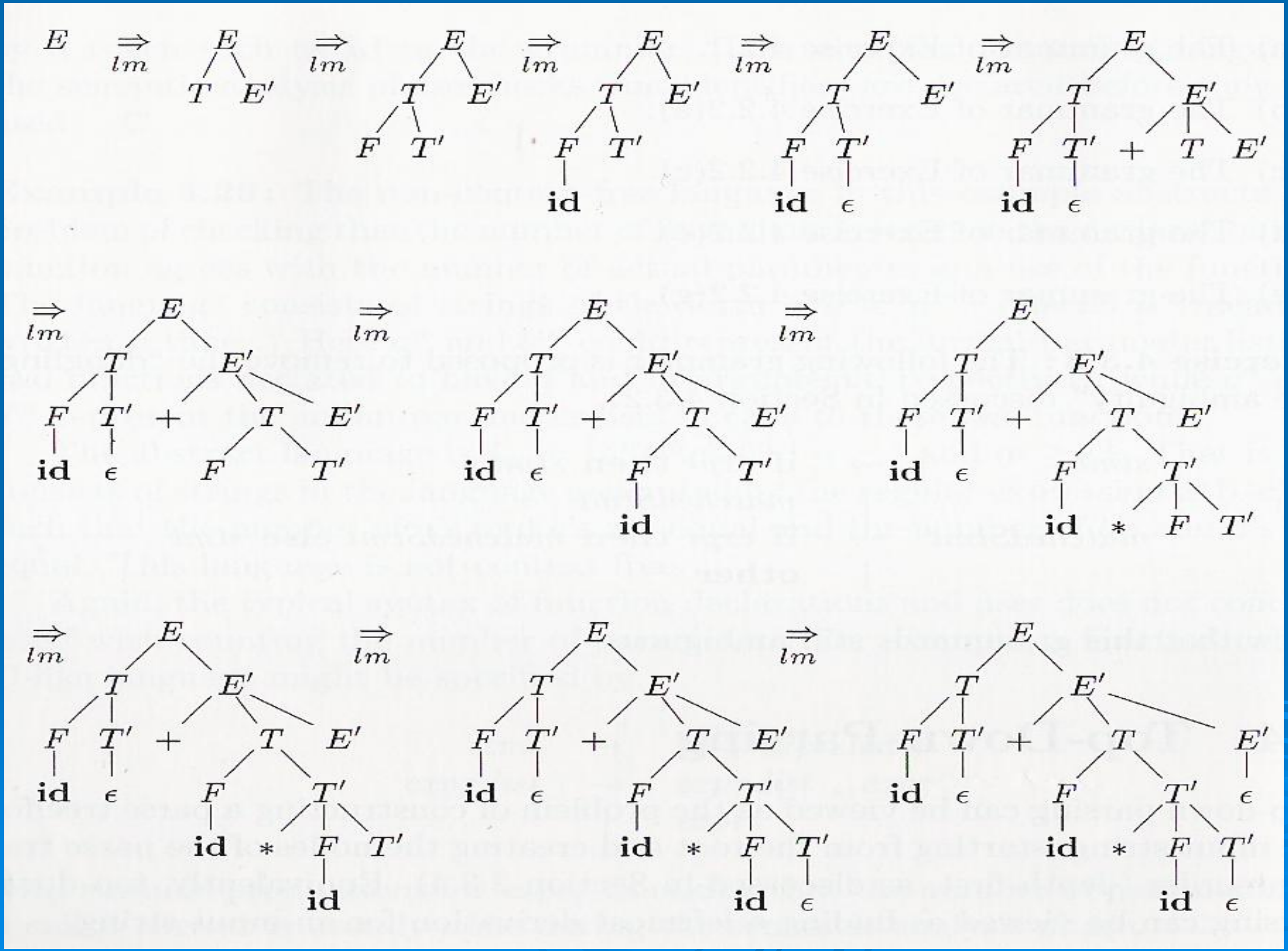
- Top-down parsing can be viewed as the problem of constructing a parse tree for the input string, starting form the root and creating the nodes of the parse tree in preorder.

- An example follows.

Creating a top-down parser (Cont.)

- Given the grammar :
  - $E \rightarrow TE'$
  - $E' \rightarrow +TE' \mid \lambda$
  - $T \rightarrow FT'$
  - $T' \rightarrow *FT' \mid \lambda$
  - $F \rightarrow (E) \mid id$
- The input: id + id * id

Creating a top-down parser (Cont.)



Top-down parsing

- A top-down parsing program consists of a set of procedures, one for each non-terminal.

- Execution begins with the procedure for the start symbol, which halts and announces success if its procedure body scans the entire input string.

Top-down parsing

A typical procedure for non-terminal A in a top-down parser:

```
boolean A() {
   choose an A-production, A → X1 X2 … Xk;
   for (i= 1 to k) {
       if (Xi is a non-terminal)
            call procedure Xi();
      else if (Xi matches the current input token “a”)
            advance the input to the next token;
      else  /* an error has occurred */;
   }
}
```

Top-down parsing

- Given a grammar:

input → expression
expression → term rest_expression
term → ID | parenthesized_expression
parenthesized_expression → '(' expression ')'
rest_expression → '+' expression | λ

Top-down parsing

- For example:

  input:

$$ID + (ID + ID)$$

Top-down parsing

Build parse tree:

start from start symbol to invoke:

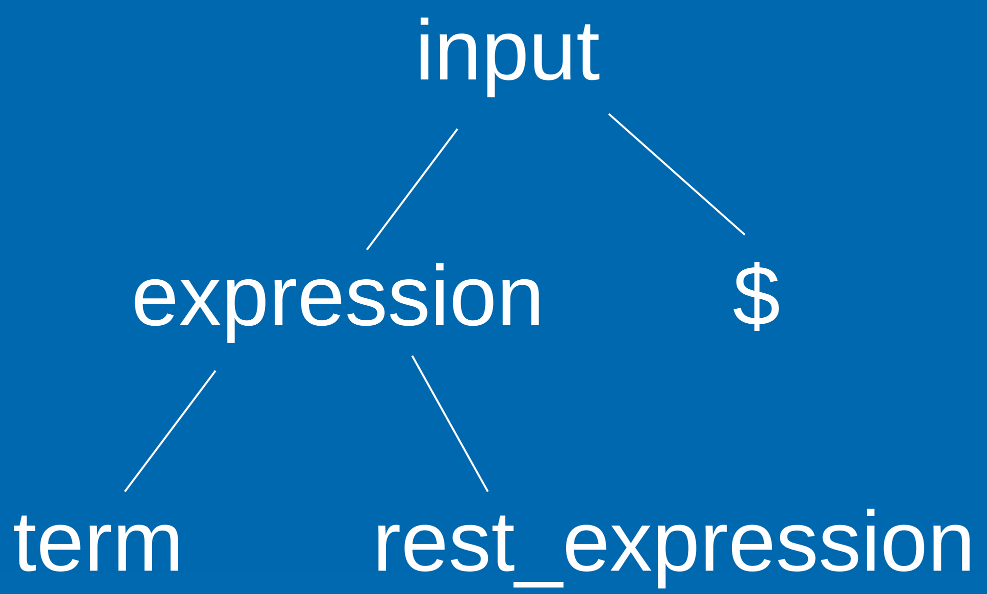
int input (void)

input

expression $

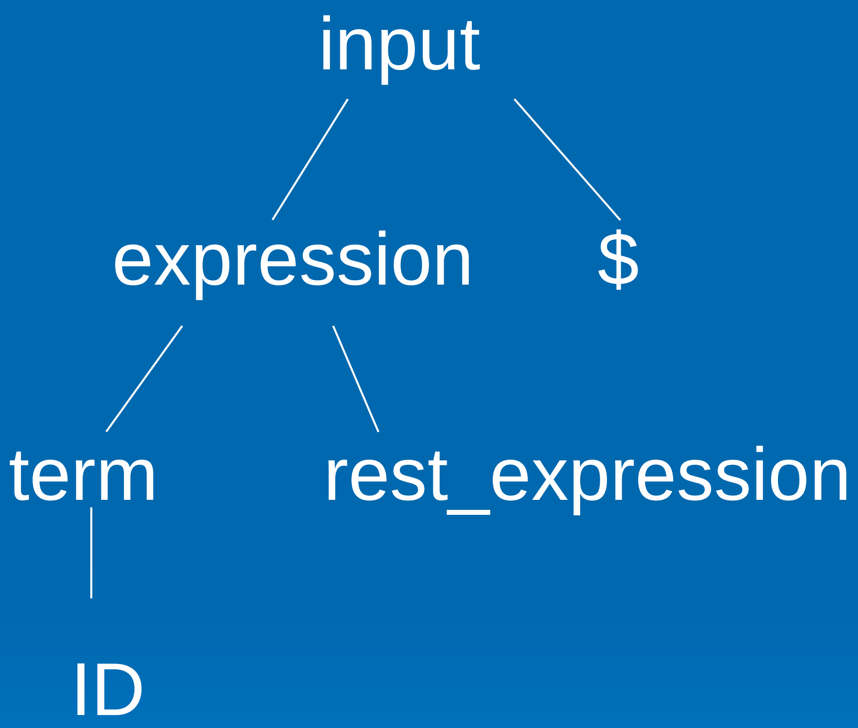
Next, invoke expression()

Top-down parsing



Next, invoke term()

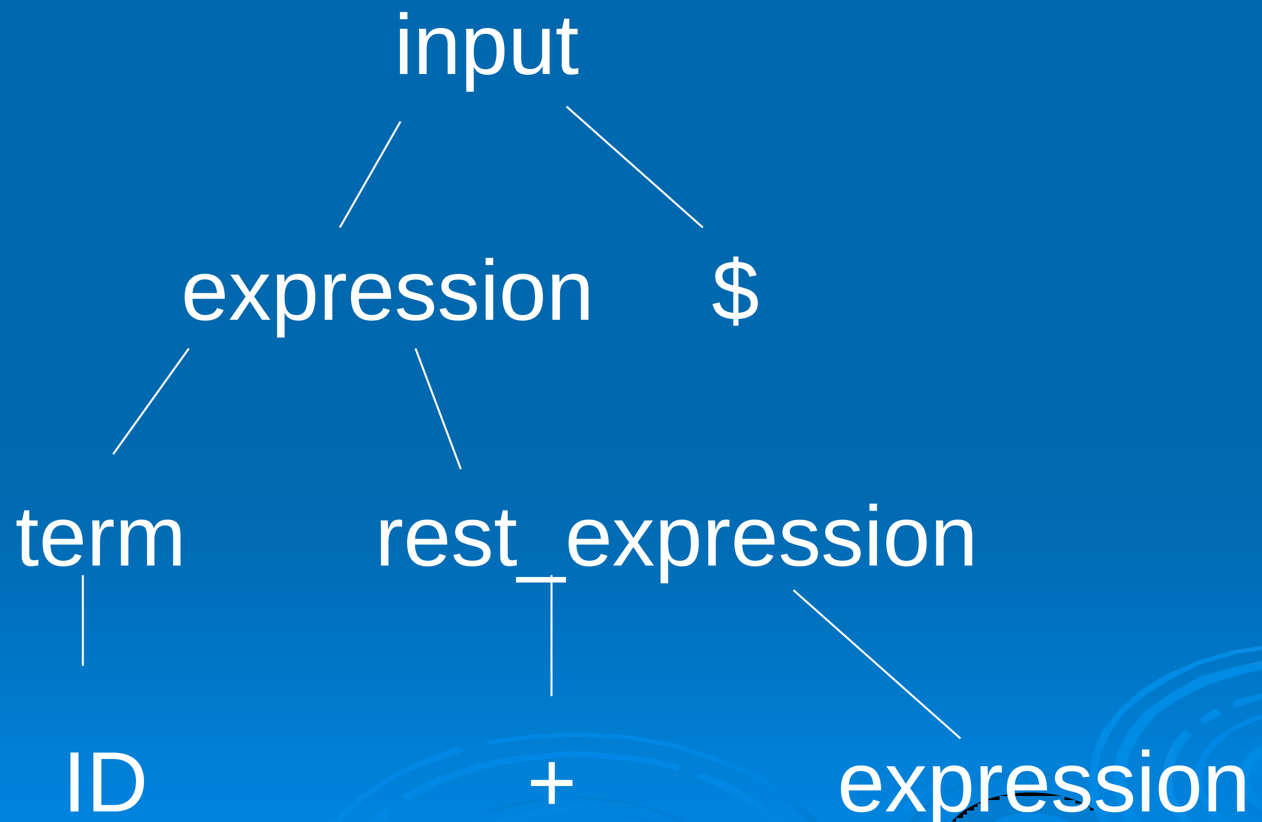
Top-down parsing



select term → ID (matching input string “ID”)

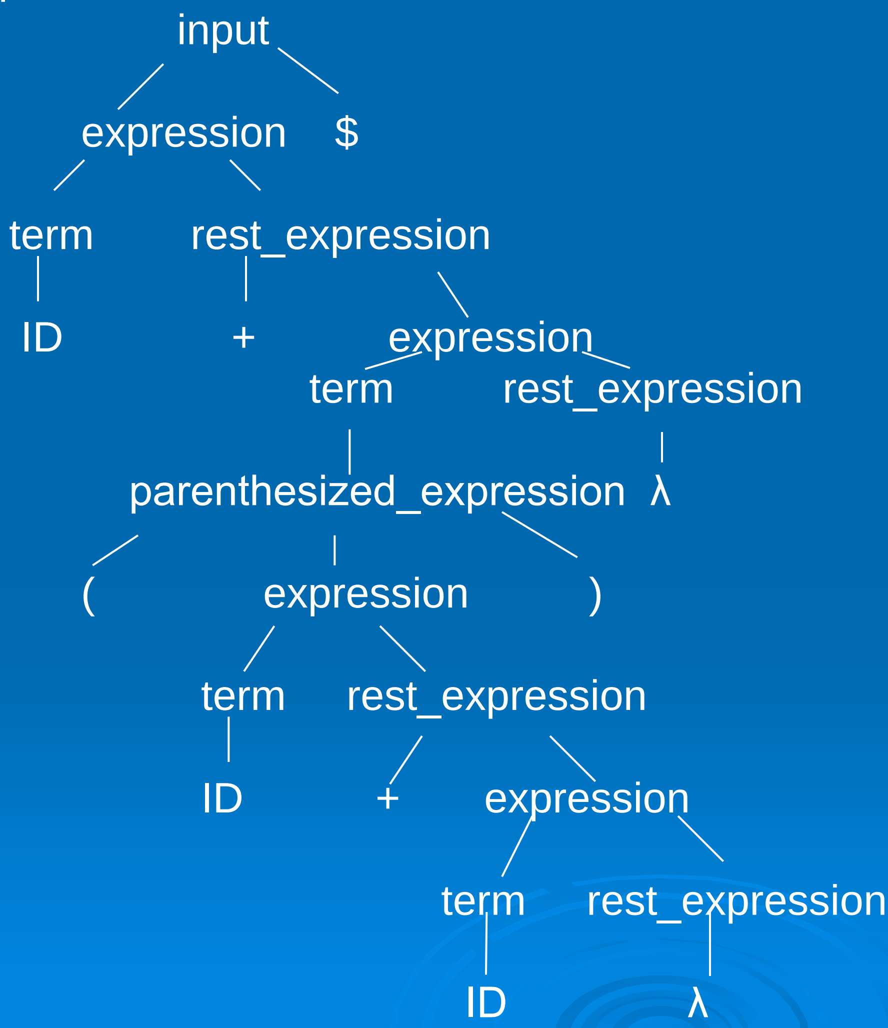
Top-down parsing

Invoke rest_expression()



Top-down parsing

The parse tree is:



LL(1) Parsers

- The class of grammars for which we can construct predictive parsers looking **k symbols ahead** in the input is called the LL(k) class.

- Predictive parsers, that is, recursive-descent parsers without backtracking, can be constructed for the LL(1) class grammars.

- The first "L" stands for scanning input from left to right. The second "L" for producing a leftmost derivation. The "1" for using one input symbol of look-ahead at each step to make parsing decisions.

LL(1) Parsers (Cont.)

$A \rightarrow \alpha \mid \beta$ are two distinct productions of grammar G, G is LL(1) if the following 3 conditions hold:

1. FIRST(α) cannot contain any terminal in FIRST(β).
2. At most one of α and β can derive λ.
3. if $\beta \rightarrow^* \lambda$, FIRST($\alpha$) cannot contain any terminal in FOLLOW(A).

   if $\alpha \rightarrow^* \lambda$, FIRST($\beta$) cannot contain any terminal in FOLLOW(A).

Nullability

- A nonterminal A is *nullable* if

$$A \Rightarrow^* \varepsilon.$$

- Clearly, A is nullable if it has a production

$$A \to \varepsilon.$$

- But A is also nullable if there are, for example, productions

$$A \to BC.$$

$$B \to A \mid aC \mid \varepsilon.$$

$$C \to aB \mid Cb \mid \varepsilon.$$

Nullability

- In other words, A is nullable if there is a production

  $$A \to \varepsilon,$$

  or there is a production

  $$A \to B_1 B_2 \ldots B_n,$$

  where B_1, B_2, ..., B_n are nullable.

Nullability

- In the grammar

$$E \to T\ E'$$

$$E' \to +\ T\ E' \mid \varepsilon.$$

$$T \to F\ T'$$

$$T' \to *\ F\ T' \mid \varepsilon.$$

$$F \to (E) \mid \textbf{id} \mid \textbf{num}$$

  E' and *T'* are nullable.

- *E*, *T*, and *F* are not nullable.

Summary

Nonterminal	Nullable
E	No
E'	Yes
T	No
T'	Yes
F	No

FIRST and FOLLOW

- Given a grammar *G*, we may define the functions FIRST and FOLLOW on the strings of symbols of *G*.
  - FIRST(α) is the set of all terminals that may appear as the *first* symbol in a replacement string of α.
  - FOLLOW(α) is the set of all terminals that may *follow* α in a derivation.

FIRST

- For a grammar symbol X, FIRST(X) is defined as follows.
  - For every terminal X, FIRST(X) = {X}.
  - For every nonterminal X, if $X \to Y_1Y_2\ldots Y_n$ is a production, then
    - FIRST(Y_1) $\subseteq$ FIRST(X).
    - Furthermore, if Y_1, Y_2, …, Y_k are nullable, then FIRST(Y_{k+1}) $\subseteq$ FIRST(X).

FIRST

- We are concerned with FIRST(X) only for the nonterminals of the grammar.
- FIRST(X) for terminals is trivial.
- According to the definition, to determine FIRST(A), we must inspect all productions that have A on the *left*.

Example: FIRST

- Let the grammar be

$$E \rightarrow T\ E'$$

$$E' \rightarrow +\ T\ E' \mid \varepsilon.$$

$$T \rightarrow F\ T'$$

$$T' \rightarrow *\ F\ T' \mid \varepsilon.$$

$$F \rightarrow (E) \mid \textbf{id} \mid \textbf{num}$$

Example: FIRST

- Find FIRST(E).
  - E occurs on the left in only one production

    $$E \to T\,E'.$$
  - Therefore, FIRST(T) $\subseteq$ FIRST(E).
  - Furthermore, T is not nullable.
  - Therefore, FIRST(E) = FIRST(T).
  - We have yet to determine FIRST(T).

Example: FIRST

- Find FIRST(T).
  - T occurs on the left in only one production

    $$T \to F\ T'.$$
  - Therefore, FIRST(F) $\subseteq$ FIRST(T).
  - Furthermore, F is not nullable.
  - Therefore, FIRST(T) = FIRST(F).
  - We have yet to determine FIRST(F).

Example: FIRST

- Find FIRST(F).
  - FIRST(F) = {(, **id**, **num**}.
- Therefore,
  - FIRST(E) = {(, **id**, **num**}.
  - FIRST(T) = {(, **id**, **num**}.

Example: FIRST

- Find FIRST(E').
  - FIRST(E') = {+}.
- Find FIRST(T').
  - FIRST(T') = {*}.

Summary

Nonterminal	Nullable	FIRST
E	No	{(, **id**, **num**}
E'	Yes	{+}
T	No	{(, **id**, **num**}
T'	Yes	{*}
F	No	{(, **id**, **num**}

FOLLOW

- For a grammar symbol X, FOLLOW(X) is defined as follows.
  - If S is the start symbol, then $\$ \in$ FOLLOW(S).
  - If $A \rightarrow \alpha B\beta$ is a production, then FIRST(β) $\subseteq$ FOLLOW(B).
  - If $A \rightarrow \alpha B$ is a production, or $A \rightarrow \alpha B\beta$ is a production and β is nullable, then FOLLOW(A) $\subseteq$ FOLLOW(B).

FOLLOW

- We are concerned about FOLLOW(X) only for the nonterminals of the grammar.
- According to the definition, to determine FOLLOW(A), we must inspect all productions that have A on the *right*.

Example: FOLLOW

- Let the grammar be

$$E \rightarrow T\ E'$$

$$E' \rightarrow +\ T\ E' \mid \varepsilon.$$

$$T \rightarrow F\ T'$$

$$T' \rightarrow *\ F\ T' \mid \varepsilon.$$

$$F \rightarrow (E) \mid \textbf{id} \mid \textbf{num}$$

Example: FOLLOW

- Find FOLLOW(E).
  - E is the start symbol, therefore $ $\in$ FOLLOW(E).
  - E occurs on the right in only one production.

$$F \to (E).$$

  - Therefore FOLLOW(E) = {$,)}.

Example: FOLLOW

- Find FOLLOW(E').
  - E' occurs on the right in two productions.

$$E \rightarrow T\ E'$$

$$E' \rightarrow +\ T\ E'.$$

  - Therefore, FOLLOW(E') = FOLLOW(E) = {\$,)}.

Example: FOLLOW

- Find FOLLOW(T).
  - T occurs on the right in two productions.

    $$E \to T\,E'$$

    $$E' \to +\,T\,E'.$$

  - Therefore, FOLLOW(T) contains FIRST(E') = {+}.
  - However, E' is nullable, therefore it also contains FOLLOW(E) = {\$,)} and FOLLOW(E') = {\$,)}.
  - Therefore, FOLLOW(T) = {+, \$,)}.

Example: FOLLOW

- Find FOLLOW(T').
  - T' occurs on the right in two productions.

$$T \to F\ T'$$

$$T' \to *\ F\ T'.$$

  - Therefore, FOLLOW(T') = FOLLOW(T) = {\$,), +}.

Example: FOLLOW

- Find FOLLOW(F).
  - F occurs on the right in two productions.

$$T \to F\ T'$$

$$T' \to *\ F\ T'.$$

  - Therefore, FOLLOW(F) contains FIRST(T') = {*}.
  - However, T' is nullable, therefore it also contains FOLLOW(T) = {+, \$,)} and FOLLOW(T') = {\$,), +}.
  - Therefore, FOLLOW(F) = {*, \$,), +}.

Summary

Nonterminal	Nullable	FIRST	FOLLOW
E	No	{(, **id**, **num**}	{$,)}
E'	Yes	{+}	{$,)}
T	No	{(, **id**, **num**}	{$,), +}
T'	Yes	{*}	{$,), +}
F	No	{(, **id**, **num**}	{*, $,), +}

Exercise

- The grammar

  $$R \to R \cup R \mid RR \mid R^* \mid (R) \mid \mathbf{a} \mid \mathbf{b}$$

  generates all regular expressions on the alphabet {**a**, **b**}.
- Using the result of the exercise from the previous lecture, find FIRST(*X*) and FOLLOW(*X*) for each nonterminal *X* in the grammar.

Construction of a predictive parsing table

- The following rules are used to construct the predictive parsing table:
  - 1. for each terminal a in FIRST(α),

    add A → α to matrix M[A,a]

  - 2. if λ is in FIRST(α), then

    for each terminal b in FOLLOW(A),

    add A → α to matrix M[A,b]

LL(1) Parsers (Cont.)

Given the grammar:

input → expression		1
expression → term rest_expression		2
term → ID		3
	| parenthesized_expression	4
parenthesized_expression → '(' expression ')'		5
rest_expression → '+' expression		6
	| λ	7

Build the parsing table.

LL(1) Parsers (Cont.)

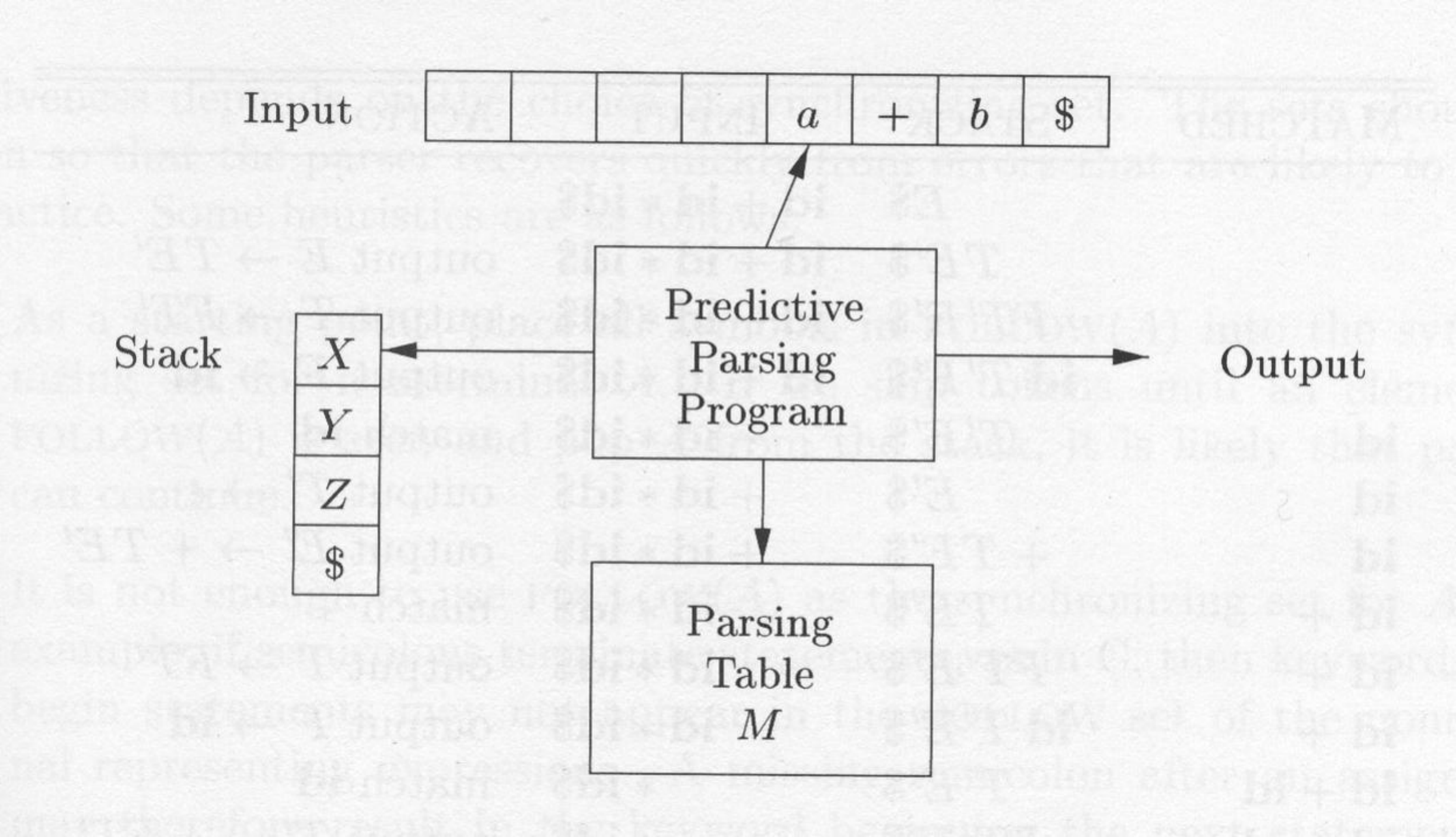
FIRST (input) = FIRST(expression)
=FIRST (term) = {ID, ‘(‘ }
FIRST (parenthesized_expression) = { ‘(‘ }
FIRST (rest_expression) = { ‘+’ λ}

FOLLOW (input) = {$ }
FOLLOW (expression) = {$ ‘)’ }
FOLLOW (term) =
FOLLOW (parenthesized_expression) = {$ ‘+’ ‘)’}
FOLLOW (rest_expression) = {$ ‘)’}

LL(1) Parsers (Cont.)

Non-terminal	Input symbol				
	ID	+	(	)	$
Input	1		1		
Expression	2		2		
Term	3		4		
parenthesized_expression			5		
rest_expression		6		7	7

Model of a table-driven predictive parser



Predictive parsing algorithm

```
Set input pointer (ip) to the first token a;
Push $ and start symbol to the stack.
Set X to the top stack symbol;
while (X != $) { /*stack is not empty*/
  if (X is token a) pop the stack and advance ip;
  else if (X is another token) error();
  else if (M[X,a] is an error entry) error();
  else if (M[X,a] = X → Y1Y2…Yk) {
          output the production X → Y1Y2…Yk;
          pop the stack;            /* pop X */
         /* leftmost derivation*/
          push Yk,Yk-1,…, Y1 onto the stack, with Y1 on top;
  }
  set X to the top stack symbol Y1;
} // end while
```

LL(1) Parsers (Cont.)

- Given the grammar:
  - $E \rightarrow TE'$ 1
  - $E' \rightarrow +TE'$ 2
  - $E' \rightarrow \lambda$ 3
  - $T \rightarrow FT'$ 4
  - $T' \rightarrow *FT'$ 5
  - $T' \rightarrow \lambda$ 6
  - $F \rightarrow (E)$ 7
  - $F \rightarrow id$ 8

LL(1) Parsers (Cont.)

FIRST(F) = FIRST(T) = FIRST(E) = { (, id }

FIRST(E') = { +, λ}

FIRST(T') = { *, λ}

FOLLOW(E) = FOLLOW(E') = {) , $ }

FOLLOW(T) = FOLLOW(T') = { +,) , $ }

FOLLOW(F) = { +, *,) , $ }

LL(1) Parsers (Cont.)

Non-terminal	Input symbols					
	Id	+	*	(	)	$
E	1			1		
E’		2			3	3
T	4			4		
T’		6	5		6	6
F	8			7		

LL(1) Parsers (Cont.)

Stack	Input	Output
$E	id + id * id $	
$E’T	id + id * id $	E → TE’
$E’T’F	id + id * id $	T → FT’
$E’T’id	id + id * id $	F → id
$E’T’	+ id * id $	match id
$E’	+ id * id $	T’ → λ
$E’T+	+ id * id $	E’ → +TE’

LL(1) Parsers (Cont.)

Stack	Input	Output
$E’T	id * id $	match +
$E’T’F	id * id $	T → FT’
$E’T’id	id * id $	F → id
$E’T’	* id $	match id
$E’T’F*	* id $	T’ → *FT’
$E’T’F	id $	match *
$E’T’id	id $	F → id
$E’T’	$	match id
$E’	$	T’ → λ
$	$	E’ → λ

Common Prefix

In Fig. 5.12(see the next slide), the common prefix:

if Expr then StmtList (R1,R2)

makes looking ahead to distinguish R1 from R2 hard.

Just use Fig. 5.13(see the next slide) to factor it and “var”(R5,6)

The resulting grammar is in Fig. 5.14.

1	Stmt	→	if Expr then StmtList endif
2		|	if Expr then StmtList else StmtList endif
3	StmtList	→	StmtList ; Stmt
4		|	Stmt
5	Expr	→	var + Expr
6		|	var

Figure 5.12: A grammar with common prefixes.

procedure F ( )
    foreach $\mathsf{A} \in N$ **do**
        $\alpha \leftarrow \mathit{LongestCommonPrefix}(\mathit{ProductionsFor}(\mathsf{A}))$
        while $|\alpha| > 0$ **do**
            $V \leftarrow$ **new** *NonTerminal* ( )
            $\mathit{Productions} \leftarrow \mathit{Productions} \cup \{\, \mathsf{A} \rightarrow \alpha V \,\}$
            foreach $p \in \mathit{ProductionsFor}(A) \mid \mathit{RHS}(p) = \alpha\beta_p$ **do** 13
                $\mathit{Productions} \leftarrow \mathit{Productions} - \{\, p \,\}$
                $\mathit{Productions} \leftarrow \mathit{Productions} \cup \{\, V \rightarrow \beta_p \,\}$
            $\alpha \leftarrow \mathit{LongestCommonPrefix}(\mathit{ProductionsFor}(\mathsf{A}))$
end

Figure 5.13: Factoring common prefixes.

1	Stmt	$\rightarrow$	if Expr then StmtList V_1
2	V_1	$\rightarrow$	endif
3		|	else StmtList endif
4	StmtList	$\rightarrow$	StmtList ; Stmt
5		|	Stmt
6	Expr	$\rightarrow$	var V_2
7	V_2	$\rightarrow$	+ Expr
8		|	λ

Figure 5.14: Factored version of the grammar in Figure 5.12.

procedure E L R ( )
    foreach $\mathsf{A} \in N$ **do**
        if $\exists\, r \in \mathit{ProductionsFor}(\mathsf{A}) \mid \mathit{RHS}(r) = \mathsf{A}\alpha$
        then
            $X \leftarrow$ **new** *NonTerminal*( )
            $Y \leftarrow$ **new** *NonTerminal*( )
            foreach $p \in \mathit{ProductionsFor}(\mathsf{A})$ **do**
                if $p = r$
                then $\mathit{Productions} \leftarrow \mathit{Productions} \cup \{\mathsf{A} \rightarrow X\,Y\}$
                else $\mathit{Productions} \leftarrow \mathit{Productions} \cup \{X \rightarrow \mathit{RHS}(p)\}$
            $\mathit{Productions} \leftarrow \mathit{Productions} \cup \{Y \rightarrow \alpha Y, Y \rightarrow \lambda\}$
end

Figure 5.15: Eliminating left recursion.

Left Recursion

- A production is **left recursive** if its LHS symbol is the first symbol of its RHS.

- In fig. 5.14, the production

  StmtList→ StmtList ; Stmt

  StmtList is left-recursion.

Left Recursion (Cont.)

1	Stmt	$\rightarrow$	if Expr then StmtList V_1
2	V_1	$\rightarrow$	endif
3		|	else StmtList endif
4	StmtList	$\rightarrow$	StmtList ; Stmt
5		|	Stmt
6	Expr	$\rightarrow$	var V_2
7	V_2	$\rightarrow$	+ Expr
8		|	λ

Figure 5.14: Factored version of the grammar in Figure 5.12.

Left Recursion (Cont.)

- Grammars with left-recursive productions can never be LL(1).
  - Some look-ahead symbol t predicts the application of the left-recursive production

    $$A \rightarrow A\beta.$$

    with **recursive-descent parsing**, the application of this production will cause

    procedure A to be invoked infinitely.

  Thus, we must eliminate left-recursion.

Left Recursion (Cont.)

Consider the following left-recursive rules.

1. $A \rightarrow A\ \alpha$
2. $\quad\quad |\ \ \beta$ the rules produce strings like $\beta\ \alpha\ \alpha$

we can change the grammar to:

1. $A \rightarrow X\ Y$
2. $X \rightarrow \beta$
3. $Y \rightarrow \alpha\ Y$
4. $\quad\quad |\ \ \lambda$ the rules also produce strings like $\beta\ \alpha\ \alpha$

The EliminateLeftRecursion algorithm is shown in fig. 5.15. Applying it to the grammar in fig. 5.14 results in fig. 5.16.

Left Recursion (Cont.)

```
procedure EliminateLeftRecursion( )
    foreach A ∈ N do
        if ∃ r ∈ ProductionsFor(A) | RHS(r) = Aα                       1
        then
            X ← new NonTerminal( )
            Y ← new NonTerminal( )
            foreach p ∈ ProductionsFor(A) do                            2
                if p = r                                                3
                then Productions ← Productions ∪ { A → X Y }
                else  Productions ← Productions ∪ { X → RHS(p) }        4
            Productions ← Productions ∪ { Y → αY, Y → λ }               5
end
```

Figure 5.15: Eliminating left recursion.

Left Recursion (Cont.)

Now, we trace the algorithm with the grammar below:

(4) StmtList → StmtList ; Stmt

(5) | Stmt

first, the input is (4) StmtList → StmtList ; Stmt

because RHS(4) = StmtList α it is left-recursive (marker 1)

create two non-terminals X, and Y

for rule (4) (marker 2)

as StmtList = StmtList,

create StmtList → XY (marker 3)

for rule (5) (marker 2)

as StmtList != Stmt

create X → Stmt (marker 4)

finally, create Y → ; Stmt and Y → λ (marker 5)

Left Recursion (Cont.)

1	Stmt	$\rightarrow$	if Expr then StmtList V_1
2	V_1	$\rightarrow$	endif
3		|	else StmtList endif
4	StmtList	$\rightarrow$	X Y
5	X	$\rightarrow$	Stmt
6	Y	$\rightarrow$	; Stmt Y
7		|	λ
8	Expr	$\rightarrow$	var V_2
9	V_2	$\rightarrow$	+ Expr
10		|	λ

Figure 5.16: LL(1) version of the grammar in Figure 5.14.

Thank you